Quantum entanglement and squeezing of the quadrature difference of bright light fields

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In this paper we present a scheme in which the amplitude-quadrature difference and phase-quadrature difference of two bright nonclassical beams serve as a pair of observed complementary variables that can be measured directly and can form a new squeezed state. Mixing such two quadrature-difference squeezed-state beams on a beam splitter produces characteristic of the Einstein-Podolsky-Rosen quantum entanglement. The experimental schemes producing such nonclassical beams and its possible applications in quantum information are discussed.

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I. INTRODUCTION

In developing quantum information science, nonlocal quantum entanglement plays a determining role [1,2]. Recently, both theoretical and experimental investigations have focused on quantum states of continuous variables in an infinite-dimensional Hilbert space, because the Einstein-Podolsky-Rosen (EPR) entangled state of light can be efficiently generated using squeezed light and beam splitters. For example, the continuously entangled EPR pairs resulting from two-mode squeezed vacuum states have been successfully employed in demonstrating unconditional quantum teleportation [3]. Two bright amplitude-squeezed sources are also proposed to be used for quantum teleportation in a relatively simpler protocol [4]. The schemes realizing highly efficient dense coding for continuous variables have been proposed, in that the two-mode squeezed-state entanglement may be utilized to achieve unconditional signal transmission [5–7]. The bright EPR beams have been experimentally produced by means of a nondegenerate optical parametric amplifier (NOPA) [8] and the Kerr nonlinearity of an optical fiber [9]. The dense coding for continuous variables based on the bright EPR beams has been experimentally demonstrated [10], in which the bright EPR beams with anticorrelation of amplitude quadrature and correlation of phase quadratures are generated from a NOPA operating in the state of deamplification and the Bell-state measurement is achieved with simple direct detection [7]. In many proposed protocols for quantum information processing using continuous light fields, a local oscillator (LO) has to be utilized for measuring the phase quadrature with homodyne detection schemes [3,11,12]. In this case, the problem of spatial and temporal mode matching increases the difficulty of the experiments and limits the detection efficiency. Especially, for bright squeezed-state light, such as from laser diodes, it is not easy to find an appropriate LO beam. Thus, sometimes, the application of bright EPR beams in quantum information is limited by the detection of phase quadrature.

Different kinds of squeezed states for arbitrary Hermitian operators have been studied and reviewed, e.g., in Ref. [13]. Two-mode and multimode first-order squeezing [14,15] have been introduced for the quadrature operator, which is defined as a linear superposition of the bosonic operators. Two-mode and multimode higher-order squeezing have been defined and investigated [16,17]. The higher order means that their "quadrature" operators are defined in terms of a product of the mode's operators. The quantum Stokes parameters provide operator representations of the polarization, which also apply to nonclassical light. Recently, the application of the quantum Stokes parameters to bright squeezed light was proposed [18] and demonstrated experimentally by a pair of spatially separated optical parametric amplifiers [19]. The specific families of the two-mode and multimode quantum states connect to the polarization state. In fact, the quantum Stokes operators are identical to the operators defined in two-mode difference squeezing [16] which is the two-mode second-order squeezing.

In this paper we extend the general theory of the two-mode first-order squeezing and utilize this type of nonclassical light in which the amplitude-quadrature difference and phase-quadrature difference of two modes are a pair of observed complementary and noncommutation variables for quantum information. When the variance of $\hat{X}_{\hat{a}_1}-\hat{X}_{\hat{a}_2}$ or $\hat{Y}_{\hat{a}_1}-\hat{Y}_{\hat{a}_2}$ of two modes falls below the quantum noise level of that of two equally coherent beams, we say, the two modes present the squeezed characteristic of quadrature differences and we call them the squeezed-state light of amplitude-quadrature or phase-quadrature difference. In the following, we will theoretically demonstrate that quadrature difference squeezed-state lights can be obtained by two bright squeezed lights, and combining two coherent quadrature-difference squeezed-state lights on a beam splitter can provide the EPR entanglement. The dependence of the squeezing and the quantum entanglement of the combined light fields on the nonclassical properties of the constituting beams are analytically discussed. Finally, the applications of the quadrature-difference squeezed-state lights in quantum teleportation and quantum dense coding are proposed.

II. OBSERVED VARIABLES OF TWO OPTICAL MODES

The two optical modes $\hat{a}_1$ and $\hat{a}_2$ have the boson commutation relations

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$$[\hat{a}_k,\hat{a}_{k'}]=[\hat{a}_k^+,\hat{a}_{k'}^+]=0,\quad [\hat{a}_k,\hat{a}_{k'}^+]=\delta_{kk'},\quad k,k'=1,2. \tag{1}$$

The quadrature phase amplitudes of the two optical modes are given by

$$\hat{X}_{\hat{a}_k}=\hat{a}_k+\hat{a}_k^+,\quad \hat{Y}_{\hat{a}_k}=-i(\hat{a}_k-\hat{a}_k^+). \tag{2}$$

The quadrature phase amplitudes obey the commutation relation

$$[\hat{X}_{\hat{a}_k},\hat{X}_{\hat{a}_{k'}}]=[\hat{Y}_{\hat{a}_k},\hat{Y}_{\hat{a}_{k'}}]=0,\quad [\hat{X}_{\hat{a}_k},\hat{Y}_{\hat{a}_{k'}}]=2i\delta_{kk'},\quad k,k'=1,2. \tag{3}$$

The unitary transformation performed on the modes $\hat{a}_1$ and $\hat{a}_2$ is given by

$$\hat{b}_1=A\hat{a}_1+B\hat{a}_2, \tag{4}$$

$$\hat{b}_2=C\hat{a}_1+D\hat{a}_2,$$

where A,B,C,D satisfy the following conditions:

$$|A|=|D| \text{ and } |B|=|C|, \tag{5}$$

$$|A|^2+|D|^2=1,$$

$$A^*C+B^*D=0.$$

It is obvious from Eqs. (4) and (5) that the operators $\hat{b}_1$, $\hat{b}_2$ satisfy the commutation relations just like modes $\hat{a}_1$ and $\hat{a}_2$,

$$[\hat{b}_k,\hat{b}_{k'}]=[\hat{b}_k^+,\hat{b}_{k'}^+]=0,\quad [\hat{b}_k,\hat{b}_{k'}^+]=\delta_{kk'}\quad k,k'=1,2. \tag{6}$$

The amplitude and phase quadratures of the operators $\hat{b}_1$, $\hat{b}_2$ also obey the commutation relation

$$[\hat{X}_{\hat{b}_k},\hat{X}_{\hat{b}_{k'}}]=[\hat{Y}_{\hat{b}_k},\hat{Y}_{\hat{b}_{k'}}]=0,\quad [\hat{X}_{\hat{b}_k},\hat{Y}_{\hat{b}_{k'}}]=2i\delta_{kk'},\quad k,k'=1,2. \tag{7}$$

In the unitary transformation $A=-B=C=D=1/\sqrt{2}$, Eqs. (4) become

$$\hat{b}_1=\frac{1}{\sqrt{2}}(\hat{a}_1-\hat{a}_2), \tag{8}$$

$$\hat{b}_2=\frac{1}{\sqrt{2}}(\hat{a}_1+\hat{a}_2).$$

The amplitude and phase quadratures of the operators $\hat{b}_1$, $\hat{b}_2$ equal

$$\hat{X}_{\hat{b}_1}=\frac{1}{\sqrt{2}}(\hat{X}_{\hat{a}_1}-\hat{X}_{\hat{a}_2}),\quad \hat{Y}_{\hat{b}_1}=\frac{1}{\sqrt{2}}(\hat{Y}_{\hat{a}_1}-\hat{Y}_{\hat{a}_2}), \tag{9}$$

$$\hat{X}_{\hat{b}_2}=\frac{1}{\sqrt{2}}(\hat{X}_{\hat{a}_1}+\hat{X}_{\hat{a}_2}),\quad \hat{Y}_{\hat{b}_2}=\frac{1}{\sqrt{2}}(\hat{Y}_{\hat{a}_1}+\hat{Y}_{\hat{a}_2}).$$

According to the commutation principle, the corresponding uncertainty relations for the quadratures are

$$\langle\delta\hat{X}^2_{\hat{b}_1}\rangle\langle\delta\hat{Y}^2_{\hat{b}_1}\rangle\geqslant 1, \tag{10}$$

$$\langle\delta\hat{X}^2_{\hat{b}_2}\rangle\langle\delta\hat{Y}^2_{\hat{b}_2}\rangle\geqslant 1.$$

If the quadratures satisfy the conditions $\langle\delta\hat{X}^2_{\hat{b}_1}\rangle\langle\delta\hat{Y}^2_{\hat{b}_2}\rangle<1$ or $\langle\delta\hat{X}^2_{\hat{b}_2}\rangle\langle\delta\hat{Y}^2_{\hat{b}_1}\rangle<1$, we call them two-mode squeezed states.

III. DIRECT DETECTION OF QUADRATURE DIFFERENCES

For simplification and without losing generality, we consider the two bright modes $\hat{a}_1$ and $\hat{a}_2$ having equal average intensities $\langle\hat{a}_1\rangle=\langle\hat{a}_2\rangle\neq 0$. A bright light field can be decomposed to a carrier $\hat{a}(0)$ oscillating at the center frequency ω_0 with an average amplitude (a_{ss}) which equals the amplitude of its steady-state field, and surrounded by "noise sidebands" $\hat{a}(\Omega)$ oscillating at a frequency of $\omega_0\pm\Omega$ with zero average amplitude

$$\langle\hat{a}(\Omega=0)\rangle=a_{ss},\quad \langle\hat{a}(\Omega\neq 0)\rangle=0. \tag{11}$$

The noise spectral component at frequency Ω is the heterodyne mixing of the carrier and the noise sidebands. The amplitude and phase quadratures are expressed by

$$\hat{X}_a(\Omega)=\hat{a}(\Omega)+\hat{a}^+(-\Omega),$$

$$\hat{Y}_a(\Omega)=\frac{1}{i}[\hat{a}(\Omega)-\hat{a}^+(-\Omega)], \tag{12}$$

with

$$[\hat{X}_a(\Omega),\hat{Y}_a(\Omega')]=2i\delta(\Omega+\Omega'). \tag{13}$$

Figure 1 shows a schematic representation of direct detection of quadrature differences of two bright modes $\hat{a}_1$ and $\hat{a}_2$ with orthogonal polarizations. The modes $\hat{a}_1$ and $\hat{a}_2$ are coherently combined on a polarizing beam splitter (PBS1) with a certainly relative phase (θ) controlled by a phase shifter and then the mixed outcome is split again into two modes $\hat{e}$ and $\hat{f}$ by PBS2. The modes $\hat{e}$ and $\hat{f}$ are

$$\hat{e}=\hat{a}_1\cos\frac{\theta}{2}+i\hat{a}_2\sin\frac{\theta}{2},$$

$$\hat{f}=\hat{a}_2\cos\frac{\theta}{2}+i\hat{a}_1\sin\frac{\theta}{2}. \tag{14}$$

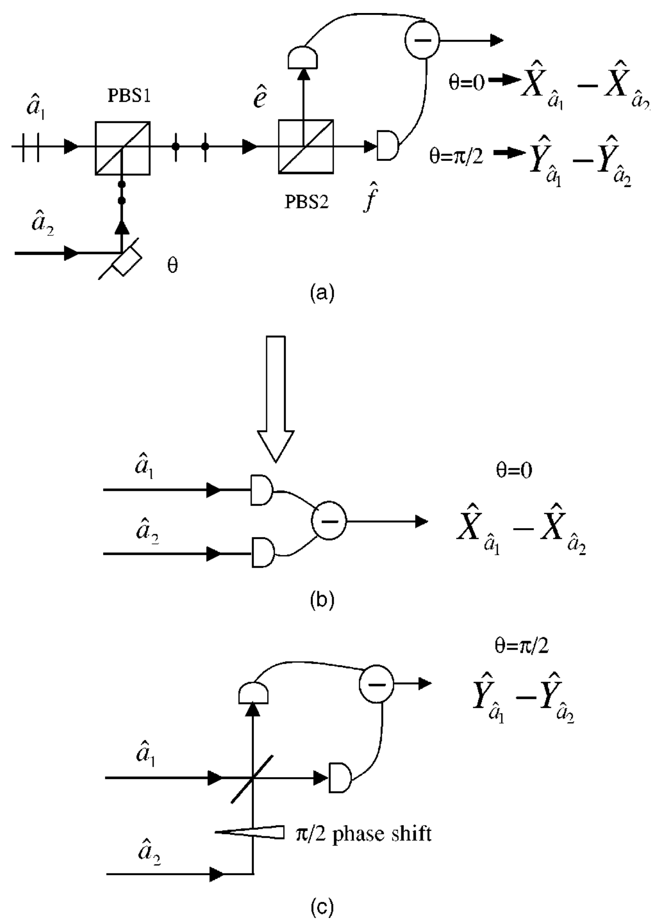

FIG. 1. The direct detection of amplitude- and phase-quadrature difference.

The bright output beams $\hat{e}$ and $\hat{f}$ are directly detected by D_1 and D_2, then the detected photocurrents are sent to subtracter as shown in Fig. 1(a).

(1) When $\theta=0$, the normalized output spectra of photocurrent difference are given by

$$\hat{i}_{-}^{0}(\Omega)=\frac{1}{\sqrt{2}}[\hat{X}_{\hat{a}_1}(\Omega)-\hat{X}_{\hat{a}_2}(\Omega)]. \tag{15}$$

This measurement is equivalent to the result of the measurement setup Fig. 1(b). The amplitude-quadrature difference measurement of two modes $\hat{a}_1$ and $\hat{a}_2$ can be achieved by direct detection.

(2) When $\theta=\pi/2$, the normalized output spectra of photocurrent difference are given by [7]

$$\hat{i}_{-}^{\pi/2}(\Omega)=\frac{1}{\sqrt{2}}[\hat{Y}_{\hat{a}_1}(\Omega)-\hat{Y}_{\hat{a}_2}(\Omega)]. \tag{16}$$

This measurement is equivalent to the result of the measurement setup Fig. 1(c) [7]. The phase-quadrature difference measurement of two modes $\hat{a}_1$ and $\hat{a}_2$ can be achieved by direct detection without the help of LO beam. Since the phase-quadrature sum of two modes cannot be measured by direct detection, we consider only amplitude- and phase-quadrature difference $\hat{X}_{\hat{a}_1-\hat{a}_2}$ and $\hat{Y}_{\hat{a}_1-\hat{a}_2}$ of two modes $\hat{a}_1$ and $\hat{a}_2$ and neglect amplitude- and phase-quadrature sum $\hat{X}_{\hat{a}_1+\hat{a}_2}$ and $\hat{Y}_{\hat{a}_1+\hat{a}_2}$ of two modes $\hat{a}_1$ and $\hat{a}_2$ as redundant variables. Thus the amplitude- and phase-quadrature difference of two modes $\hat{a}_1$ and $\hat{a}_2$ will exhibit a squeezed state just like the amplitude and phase of a single mode. When the normalized variances of the quadrature differences satisfy the inequalities

$$\begin{aligned}\langle\delta\hat{X}^2_{\hat{a}_1-\hat{a}_2}\rangle&=\left\langle\delta\left(\frac{1}{\sqrt{2}}[\hat{X}_{\hat{a}_1}(\Omega)-\hat{X}_{\hat{a}_2}(\Omega)]\right)^2\right\rangle\\&<1<\langle\delta\hat{Y}^2_{\hat{a}_1-\hat{a}_2}\rangle\\&=\left\langle\delta\left(\frac{1}{\sqrt{2}}[\hat{Y}_{\hat{a}_1}(\Omega)-\hat{Y}_{\hat{a}_2}(\Omega)]\right)^2\right\rangle\end{aligned}$$

or

$$\begin{aligned}&\left\langle\delta\left(\frac{1}{\sqrt{2}}[\hat{Y}_{\hat{a}_1}(\Omega)-\hat{Y}_{\hat{a}_2}(\Omega)]\right)^2\right\rangle\\&\quad<1<\left\langle\delta\left(\frac{1}{\sqrt{2}}[\hat{X}_{\hat{a}_1}(\Omega)-\hat{X}_{\hat{a}_2}(\Omega)]\right)^2\right\rangle,\end{aligned}$$

we say, they constitute an amplitude-quadrature difference or a phase-quadrature difference squeezed-state light field. For generating the quadrature-difference squeezed-state lights, modes $\hat{a}_1$ and $\hat{a}_2$ may be two bright amplitude-quadrature or phase-quadrature squeezed lights, and also quantum correlated EPR beams such as the signal and idler modes produced by a NOPA, which will be discussed in detail later. In the quadrature-difference squeezed-state lights the anti-squeezed complementary components with large noise are, respectively, the phase-quadrature difference for the amplitude-quadrature difference squeezing and the amplitude-quadrature difference for the phase-quadrature difference squeezing.

IV. GENERATION OF EPR ENTANGLED BEAMS USING QUADRATURE-DIFFERENCE SQUEEZED-STATE LIGHTS

Combining two quadrature-difference squeezed-state lights with classical coherence on two beam splitters (BS1 and BS2) we can obtain EPR entangled beams. The schematic representation is shown in Fig. 2. The amplitude- and phase-quadrature differences of output beams from beam splitters BS1 and BS2 are given by

$$\hat{X}_{\hat{c}_1-\hat{c}_2}=\frac{1}{2}(\hat{X}_{\hat{a}_1-\hat{a}_2}-\hat{Y}_{\hat{a}_1-\hat{a}_2}+\hat{Y}_{\hat{a}_3-\hat{a}_4}+\hat{X}_{\hat{a}_3-\hat{a}_4}), \tag{17}$$

$$\hat{Y}_{\hat{c}_1-\hat{c}_2}=\frac{1}{2}(\hat{Y}_{\hat{a}_1-\hat{a}_2}-\hat{X}_{\hat{a}_1-\hat{a}_2}+\hat{X}_{\hat{a}_3-\hat{a}_4}+\hat{Y}_{\hat{a}_3-\hat{a}_4}),$$

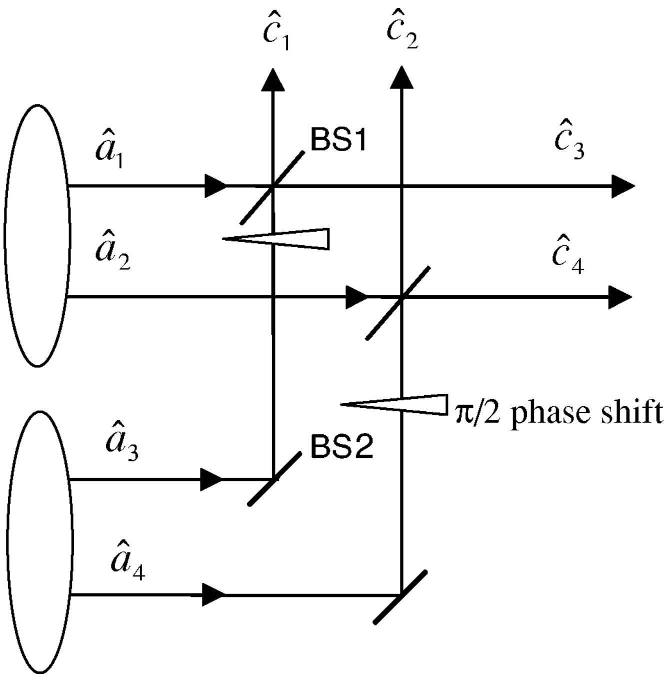

FIG. 2. EPR entanglement produced by two amplitude- (phase-) quadrature difference squeezed states.

$$\hat{X}_{\hat{c}_3-\hat{c}_4}=\frac{1}{2}(\hat{X}_{\hat{a}_1-\hat{a}_2}+\hat{Y}_{\hat{a}_1-\hat{a}_2}-\hat{Y}_{\hat{a}_3-\hat{a}_4}+\hat{X}_{\hat{a}_3-\hat{a}_4}),$$

$$\hat{Y}_{\hat{c}_3-\hat{c}_4}=\frac{1}{2}(\hat{Y}_{\hat{a}_1-\hat{a}_2}+\hat{X}_{\hat{a}_1-\hat{a}_2}-\hat{X}_{\hat{a}_3-\hat{a}_4}+\hat{Y}_{\hat{a}_3-\hat{a}_4}).$$

When input beams are amplitude difference squeezed states $\langle\delta\hat{X}^2_{\hat{a}_1-\hat{a}_2}\rangle<1$ and $\langle\delta\hat{X}^2_{\hat{a}_3-\hat{a}_4}\rangle<1$, the output beam pairs $\hat{c}_1$, $\hat{c}_2$, and $\hat{c}_3$, $\hat{c}_4$ have anticorrelated amplitude-quadrature difference $[\langle\delta(\hat{X}_{\hat{c}_1-\hat{c}_2}+\hat{X}_{\hat{c}_3-\hat{c}_4})^2\rangle$ is below the corresponding standard quantum limit (SQL)] and correlated phase-quadrature difference $[\langle\delta(\hat{Y}_{\hat{c}_1-\hat{c}_2}-\hat{Y}_{\hat{c}_3-\hat{c}_4})^2\rangle<\text{SQL}]$. Conversely, when input beams are phase-quadrature difference squeezed states $\langle\delta\hat{Y}^2_{\hat{a}_1-\hat{a}_2}\rangle<1$ and $\langle\delta\hat{Y}^2_{\hat{a}_3-\hat{a}_4}\rangle<1$, the output beam pairs have correlation of the amplitude-quadrature difference $[\langle\delta(\hat{X}_{\hat{c}_1-\hat{c}_2}-\hat{X}_{\hat{c}_3-\hat{c}_4})^2\rangle<\text{SQL}]$ and anticorrelated phase-quadrature difference $[\langle\delta(\hat{Y}_{\hat{c}_1-\hat{c}_2}+\hat{Y}_{\hat{c}_3-\hat{c}_4})^2\rangle<\text{SQL}]$.

For experiments, the beam pairs ($\hat{a}_1$, $\hat{a}_2$ and $\hat{a}_3$, $\hat{a}_4$) of quadrature-difference squeezed-state lights can be produced, for example, from two NOPAs with identical configuration pumped by a same laser source [20,8,10]. Two coherent input signals $a_\updownarrow$ and $a_\leftrightarrow$ with the same frequency ω_0 and orthogonal polarization are injected into a NOPA. For simplification, the polarizations of the injected signal and idler field are orientated along the vertical and horizontal directions, and their intensities and original phases before NOPA are considered to be identical. The amplifier is pumped with the second-harmonic wave of $\omega_p=2\omega_0$ and amplitude of pump field $a_p\gg a_\updownarrow$, $a_\leftrightarrow$; in this case the pump field can be considered as a classical field without depletion during the amplification process. The output signal and idler fields polarized along the vertical and horizontal direction are denoted with $b_\updownarrow$ and $b_\leftrightarrow$. The input-output Heisenberg evolutions of the field modes in NOPA are given by [21]

$$\hat{b}_{0\updownarrow}=\mu\hat{a}_{0\updownarrow}+\nu\hat{a}^+_{0\leftrightarrow},\quad \hat{b}_{0\leftrightarrow}=\mu\hat{a}_{0\leftrightarrow}+\nu\hat{a}^+_{0\updownarrow},\tag{18}$$

$$\hat{b}_{+\updownarrow}=\mu\hat{a}_{+\updownarrow}+\nu\hat{a}^+_{+\leftrightarrow},\quad \hat{b}_{+\leftrightarrow}=\mu\hat{a}_{+\leftrightarrow}+\nu\hat{a}^+_{+\updownarrow},$$

$$\hat{b}_{-\updownarrow}=\mu\hat{a}_{-\updownarrow}+\nu\hat{a}^+_{-\leftrightarrow},\quad \hat{b}_{-\leftrightarrow}=\mu\hat{a}_{-\leftrightarrow}+\nu\hat{a}^+_{-\updownarrow},$$

where $\hat{a}$, $\hat{a}^+$ and $\hat{b}$, $\hat{b}^+$ denote the annihilation and creation operators of the input and the output modes The subscripts 0 and $\pm$ stand for the central mode at frequency ω_0 and the sidebands at frequency $\omega_0\pm\Omega$, respectively. The parameters $\mu=\cosh r$ and $\nu=e^{i\theta_p}\sinh r$ are the function of the squeezing factor $r(r\propto L\chi^2|a_p|$, L is the nonlinear crystal length, χ^2 is the effective second-order susceptibility of the nonlinear crystal in NOPA, and a_p is the amplitude of the pump field) and the phase θ_p of the pump field. In the following calculation the phase θ_p is set to zero as the reference of relative phase of all other light fields. For a bright optical field, the quadratures of the output orthogonal polarization modes at a certain rotated phase θ are expressed by

$$\hat{X}_{\hat{b}_\updownarrow}(\theta)=\frac{b^*_{0\updownarrow}\hat{b}_{+\updownarrow}e^{-i\theta}+b_{0\updownarrow}\hat{b}^+_{-\updownarrow}e^{i\theta}}{|b_{0\updownarrow}|}=\hat{b}_{+\updownarrow}e^{-i(\theta+\varphi)}+\hat{b}^+_{-\updownarrow}e^{i(\theta+\varphi)},\tag{19}$$

$$\hat{X}_{\hat{b}_\leftrightarrow}(\theta)=\hat{b}_{+\leftrightarrow}e^{-i(\theta+\varphi)}+\hat{b}^+_{-\leftrightarrow}e^{i(\theta+\varphi)},$$

where $\varphi=\arg(b_{0\updownarrow})=\arg(b_{0\leftrightarrow})=\arg(e^{i\Phi}+e^{-i\Phi}\tanh r)$ is the phase of the modes $\hat{b}_{0\updownarrow}$, $\hat{b}_{0\leftrightarrow}$ relative to θ_p and Φ is the phase of the modes $\hat{a}_{0\updownarrow}$, $\hat{a}_{0\leftrightarrow}$ relative to θ_p. Taking $\theta=0$ and $\theta=\pi/2$ in Eq. (19), the amplitude and phase quadratures of the output fields are obtained,

$$\hat{X}_{\hat{b}_\updownarrow}=\hat{X}_{\hat{b}_\updownarrow}(0)=\hat{b}_{+\updownarrow}e^{-i\varphi}+\hat{b}^+_{-\updownarrow}e^{i\varphi},\tag{20}$$

$$\hat{X}_{\hat{b}_\leftrightarrow}=\hat{X}_{\hat{b}_\leftrightarrow}(0)=\hat{b}_{+\leftrightarrow}e^{-i\varphi}+\hat{b}^+_{-\leftrightarrow}e^{i\varphi},$$

$$\hat{Y}_{\hat{b}_\updownarrow}=\hat{X}_{\hat{b}_\updownarrow}\left(\frac{\pi}{2}\right)=-i(\hat{b}_{+\updownarrow}e^{-i\varphi}-\hat{b}^+_{-\updownarrow}e^{i\varphi}),$$

$$\hat{Y}_{\hat{b}_\leftrightarrow}=\hat{X}_{\hat{b}_\leftrightarrow}\left(\frac{\pi}{2}\right)=-i(\hat{b}_{+\leftrightarrow}e^{-i\varphi}-\hat{b}^+_{-\leftrightarrow}e^{i\varphi}).$$

When the injected subharmonic signal and harmonic pump field are in phase $(\Phi=\varphi=0)$, the maximum parametric amplification is achieved [6]. The difference of the amplitude quadratures and the difference of the phase quadratures between two orthogonal polarization modes are

$$\hat{X}_{\hat{b}_\updownarrow}-\hat{X}_{\hat{b}_\leftrightarrow}=e^{-r}\hat{X}_{\hat{a}_\updownarrow}-e^{-r}\hat{X}_{\hat{a}_\leftrightarrow},\tag{21}$$

$$\hat{Y}_{\hat{b}_\updownarrow}-\hat{Y}_{\hat{b}_\leftrightarrow}=e^{r}\hat{Y}_{\hat{a}_\updownarrow}+e^{r}\hat{Y}_{\hat{a}_\leftrightarrow}.$$

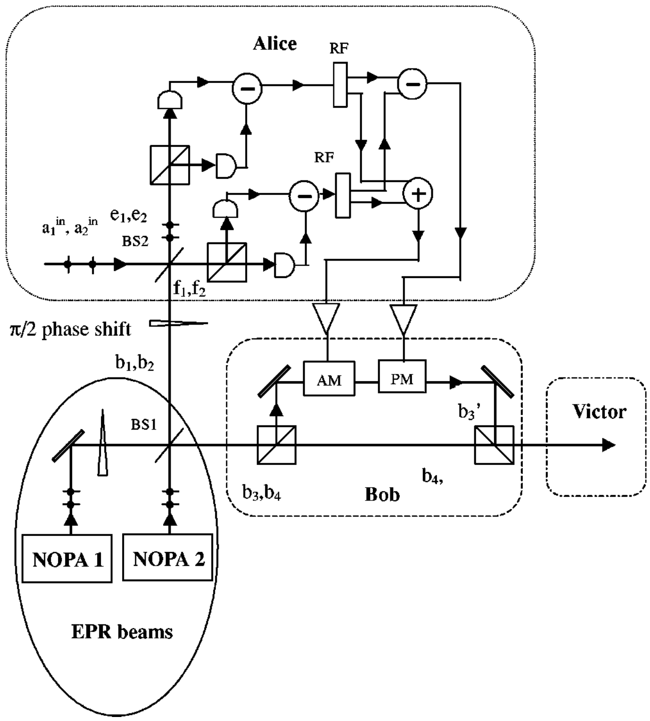

FIG. 3. A schematic representation of quantum teleportation using quadrature-difference squeezing.

When the input modes $\hat{a}_{\updownarrow}$, $\hat{a}_{\leftrightarrow}$ of NOPA are the coherent state, $\langle \delta\hat{X}^2_{\hat{a}_{\updownarrow}}\rangle=\langle \delta\hat{X}^2_{\hat{a}_{\leftrightarrow}}\rangle=\langle \delta\hat{Y}^2_{\hat{a}_{\updownarrow}}\rangle=\langle \delta\hat{Y}^2_{\hat{a}_{\leftrightarrow}}\rangle=1$, we can readily write the variances of amplitude and phase quadrature difference,

$$\langle \delta\hat{X}^2_{\hat{b}_{\updownarrow}-\hat{b}_{\leftrightarrow}}\rangle=e^{-2r},\ \ \langle \delta\hat{Y}^2_{\hat{b}_{\updownarrow}-\hat{b}_{\leftrightarrow}}\rangle=e^{2r}.$$

When $r>0$, the output orthogonal polarization modes are an amplitude-difference squeezed state. When the injected subharmonic signal and harmonic pump field are out of phase, i.e., $\Phi=\varphi=\pi/2$, NOPA operates at parametric deamplification [10,22]. Therefore the amplitude-quadrature and the phase-quadrature differences are

$$\hat{X}_{\hat{b}_{\updownarrow}}-\hat{X}_{\hat{b}_{\leftrightarrow}}=e^{r}\hat{Y}_{\hat{a}_{\updownarrow}}+e^{r}\hat{Y}_{\hat{a}_{\leftrightarrow}}, \tag{22}$$

$$\hat{Y}_{\hat{b}_{\updownarrow}}-\hat{Y}_{\hat{b}_{\leftrightarrow}}=e^{-r}\hat{X}_{\hat{a}_{\updownarrow}}-e^{-r}\hat{X}_{\hat{a}_{\leftrightarrow}}.$$

Obviously, the squeezed state of phase-quadrature difference is obtained for $r>0$.

V. APPLICATION OF QUADRATURE-DIFFERENCE SQUEEZED-STATE BEAMS IN QUANTUM INFORMATION

An important example in quantum communication is quantum teleportation, which is the disembodied transport of an unknown quantum state from one place to another. The diagram for teleportation of continuous variable using the EPR correlation of amplitude- and phase-quadrature difference is shown in Fig. 3. We assume that the two NOPAs are operated at deamplification. The outputs of two NOPAs are mixed on a 50% beam splitter (BS1) to generate the EPR beams. A half of the EPR beams $\hat{b}_1$, $\hat{b}_2$ is sent to the sender (Alice) where it is mixed with the input signal beams $\hat{a}_1^{in}$, $\hat{a}_2^{in}$ which have same intensity as $\hat{b}_1$, $\hat{b}_2$ and $\pi/2$ phase shift on the second 50% beam splitter (BS2). The bright output beams, $\hat{e}_1$, $\hat{e}_2$ and $\hat{f}_1$, $\hat{f}_2$, from BS2 are given by

$$\hat{e}_1=\frac{\sqrt{2}}{2}(\hat{a}_1^{in}+i\hat{b}_1), \tag{23}$$

$$\hat{e}_2=\frac{\sqrt{2}}{2}(\hat{a}_2^{in}+i\hat{b}_2),$$

$$\hat{f}_1=\frac{\sqrt{2}}{2}(\hat{a}_1^{in}-i\hat{b}_1),$$

$$\hat{f}_2=\frac{\sqrt{2}}{2}(\hat{a}_2^{in}-i\hat{b}_2).$$

The amplitude-quadrature differences of bright output beams, $\hat{e}_1$, $\hat{e}_2$ and $\hat{f}_1$, $\hat{f}_2$ are directly detected using the method shown in Fig. 1. Each of the output photocurrents from subtracter is divided into two parts by a rf power splitter. The sum and difference of the divided photocurrents are expressed by

$$\hat{i}_+=\frac{1}{2}[(\hat{X}_{\hat{a}_1^{in}}-\hat{X}_{\hat{a}_2^{in}})+(\hat{X}_{\hat{b}_1}-\hat{X}_{\hat{b}_2})], \tag{24}$$

$$\hat{i}_-=\frac{1}{2}[(\hat{Y}_{\hat{a}_1^{in}}-\hat{Y}_{\hat{a}_2^{in}})-(\hat{Y}_{\hat{b}_1}-\hat{Y}_{\hat{b}_2})].$$

Thus the joint measurement between a half of the EPR beams and input signal beams is achieved with this simple direct detection. Then the photocurrents $\hat{i}_+$ and $\hat{i}_-$ are sent to amplitude and phase modulator in the receiver (Bob), respectively. The amplitude and phase modulators transform the photocurrents into beams $\hat{b}_3$. The output beam $\hat{b}_3'$ from modulators is expressed by

$$\hat{b}_3'=\hat{b}_3+g_+\hat{i}_++g_-\hat{i}_-, \tag{25}$$

where g_+ and g_- describe Bob's (suitably normalized) amplitude and phase gains for the transformation from photocurrent to output beam. The output amplitude- and phase-quadrature difference are given by

$$\hat{X}_{\hat{b}_3'-\hat{b}_4}=\hat{X}_{\hat{a}_1^{in}-\hat{a}_2^{in}}+(\hat{X}_{\hat{b}_1-\hat{b}_2}+\hat{X}_{\hat{c}_3-\hat{c}_4}), \tag{26}$$

$$\hat{Y}_{\hat{b}_3'-\hat{b}_4}=\hat{Y}_{\hat{a}_1^{in}-\hat{a}_2^{in}}+(\hat{Y}_{\hat{b}_1-\hat{b}_2}-\hat{Y}_{\hat{c}_3-\hat{c}_4}),$$

where $g_+=g_-=\sqrt{2}$. Under ideal conditions, $\langle \delta(\hat{X}_{\hat{b}_1-\hat{b}_2}+\hat{X}_{\hat{c}_3-\hat{c}_4})^2\rangle\rightarrow 0$ and $\langle \delta(\hat{Y}_{\hat{b}_1-\hat{b}_2}-\hat{Y}_{\hat{c}_3-\hat{c}_4})^2\rangle\rightarrow 0$, we obtain from Eq. (26),

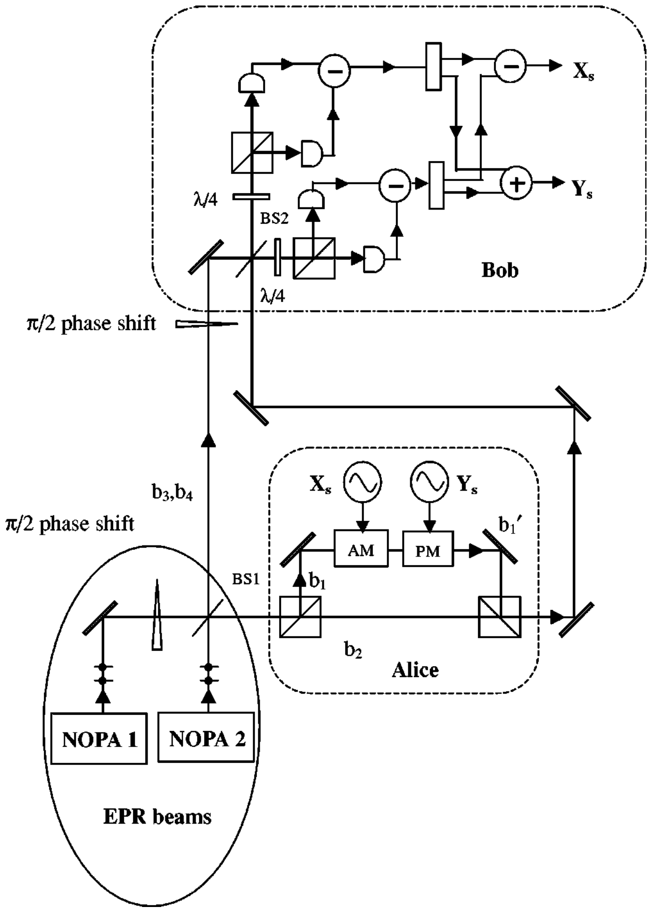

FIG. 4. Schematic representation of dense coding.

$$\hat{X}_{\hat{b}_3'-\hat{b}_4}=\hat{X}_{\hat{a}_1^{in}-\hat{a}_2^{in}}, \tag{27}$$

$$\hat{Y}_{\hat{b}_3'-\hat{b}_4}=\hat{Y}_{\hat{a}_1^{in}-\hat{a}_2^{in}}.$$

Thus we obtain quantum teleportation of two modes difference. It is easy to demonstrate quantum teleportation at Victor using direct measurement.

A scheme to demonstrate the dense coding by means of the EPR correlation of amplitude- and phase-quadrature difference is shown in Fig. 4. We use two NOPAs operating at amplification. The outputs of two NOPAs are mixed on a 50% beam splitter (BS1) to generate the EPR beams. A half of the EPR beams $\hat{b}_1$, $\hat{b}_2$ is sent to the sender (Alice) and the classical amplitude and phase signals are modulated on beam $\hat{b}_1$, which leads to a displacement of a_s,

$$\hat{b}_1'=\hat{b}_1+a_s, \tag{28}$$

where a_s is the signal sent via the quantum channel. From Eqs. (17) and (22) we know that there are very large noises $\langle \delta\hat{X}_{\hat{b}_1}{}^2\rangle\to\infty$, $\langle \delta\hat{Y}_{\hat{b}_1}{}^2\rangle\to\infty$, $\langle \delta\hat{X}^2_{\hat{b}_1-\hat{b}_2}\rangle\to\infty$, $\langle \delta\hat{Y}^2_{\hat{b}_1-\hat{b}_2}\rangle\to\infty$ for perfect EPR beams, therefore, the signal-to-noise ratio $\mathcal{R}$ tends to zero,

$$\mathcal{R}_{X_s}=\frac{\langle \delta X_s^2\rangle}{\langle \delta\hat{X}_{\hat{b}_1}{}^2\rangle}=\frac{\langle \delta X_s^2\rangle}{\langle \delta\hat{X}_{\hat{b}_1-\hat{b}_2}{}^2\rangle}\to 0, \tag{29}$$

$$\mathcal{R}_{Y_s}=\frac{\langle \delta Y_s^2\rangle}{\langle \delta\hat{Y}_{\hat{b}_1}{}^2\rangle}=\frac{\langle \delta Y_s^2\rangle}{\langle \delta\hat{Y}_{\hat{b}_1-\hat{b}_2}{}^2\rangle}\to 0.$$

No one other than Bob can attain any information of the signal from the modulated half of EPR beams in ideal conditions because the signal is submerged in large noises. The signal can only be demodulated with the aid of the other half of EPR beams ($\hat{b}_3$, $\hat{b}_4$) which is quantum correlated with the modulated half ($\hat{b}_1'$, $\hat{b}_2$). At Bob, the beams ($\hat{b}_3$, $\hat{b}_4$) are combined with the modulated half ($\hat{b}_1'$, $\hat{b}_2$) in a relative phase of $\pi/2$ on the 50% beam splitter (BS2). The phase-quadrature differences of bright output beams are directly detected using the method shown in Fig. 1. Each of the output photocurrents of subtracter is divided into two parts by the rf power splitter. The sum and difference of the divided photocurrents are expressed by

$$\hat{i}_+=\frac{1}{2}[(\hat{Y}_{\hat{b}_1}-\hat{Y}_{\hat{b}_2})+(\hat{Y}_{\hat{b}_3}-\hat{Y}_{\hat{b}_4})]+\frac{1}{2}X_s=\frac{1}{2}X_s, \tag{30}$$

$$\hat{i}_-=\frac{1}{2}[(\hat{X}_{\hat{b}_1}-\hat{X}_{\hat{b}_2})-(\hat{X}_{\hat{b}_3}-\hat{X}_{\hat{b}_4})]+\frac{1}{2}Y_s=\frac{1}{2}Y_s,$$

where $\langle \delta(\hat{X}_{\hat{b}_1-\hat{b}_2}-\hat{X}_{\hat{b}_3-\hat{b}_4})^2\rangle\to 0$, $\langle \delta(\hat{Y}_{\hat{b}_1-\hat{b}_2}-\hat{Y}_{\hat{b}_3-\hat{b}_4})^2\rangle\to 0$ in ideal conditions. Thus we can extract simultaneously the amplitude- and phase-quadrature informations modulated on the signal beam with a sensitivity beyond the SQL using a simple direct detection system.

VI. CONCLUSION

In conclusion, we have shown that the amplitude-quadrature difference and phase-quadrature difference of two light fields may serve as a pair of observed complementary variables that can be directly detected. The experimental schemes obtaining the quadrature-difference squeezed-state lights and the EPR entanglement are proposed. Possible applications of this type of EPR correlated beams in quantum information have been discussed. Usability of a simple direct detection system for Bell state is the significant advantage of the proposed schemes. Since the local oscillator is not needed, the mode-mismatching trouble encountered in experiments with homodyne detection is overcome. We provide a different way from the quantum Stokes parameters to understand and apply quadrature-difference squeezing of two bright light fields. Moreover, our theory may be generalized easily to multimode squeezing.

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